نظرية المعلومات والترميز - امتحان ثاني

Specialization:	Electrical Engineering	PTU (Palestine Technical University logo)	Palestinian National Authority Ministry Education & Higher Education Palestine Technical University College of Engineering & Technology
Course Name:	Information Theory and Coding		
Date:	14/04/2011		Second Exam Second semester 2010/2011
Time:	11:00-12:00		
Instructor:	Dr. Mutamed Khatib		
Name:	[redacted] Answer key	Section:.	/20

Answer *all* the following 4 questions

Q1. A $(7,4)$ block code has a parity check matrix:

$$\mathbf{H}=\begin{bmatrix}1&1&0&1&1&0&0\\1&1&1&0&0&1&0\\1&0&1&1&0&0&1\end{bmatrix}$$

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This code can correct one error.

a. *(2 marks)* Derive the generator matrix of this code and encode the data 1110

b. *(2 marks)* Derive a syndrome decoding for the code as described above and decode the received data 1101110

~~c. *(2 marks)* Calculate the maximum number of errors a $(15,11)$ block code can correct.~~

$D_{min}=3$

Q2. *(10marks)* For a $(6,3)$ systematic linear block code, the 3 parity check digits are:

$$P_1 = 1\times I_1 \oplus 1\times I_2 \oplus 1\times I_3$$
$$P_2 = 1\times I_1 \oplus 1\times I_2 \oplus 0\times I_3$$
$$P_3 = 0\times I_1 \oplus 1\times I_2 \oplus 1\times I_3$$

a. *(2 marks)* Construct the generator matrix for this code.

b. *(2 marks)* Construct all possible codewords generated by this matrix

c. *(2 marks)* Determine the error correcting capabilities for this code

d. *(2 marks)* Prepare a suitable decoding table

e. *(2 marks)* Decode the received words: 101100.

Q3. *(4marks)* When generating a $(7,4)$ cyclic block code using the polynomial x^3+x^2+1:

a. *(2 marks)* What would the generated codeword be for the data sequence 1000?

b. *(1 marks)* Check that this codeword would produce a zero syndrome if received without errors.

c. *(1 marks)* If the codeword 1000110 is corrupted to 1001110, what is the syndrome at the receiver?

تم الرفع بواسطة
م. معن أبوعيسى

Good luck

Q.1

(a) G = [1 1 0 1 0 (struck out)

1 1 0

0 1 1

1 0 1]

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$ (2)

d = 1110 ⇒ c = dG = [1110 010] (1)

(b)

0 0 0 0 0 0 0		0 0 0
1 0 0 0 0 0 0		0 1 1
0 1 0 0 0 0 0		0 1 0
0 0 1 0 0 0 0		0 1 0
0 0 0 1 0 0 0		1 0 1
0 0 0 0 1 0 0		0 0 0
0 0 0 0 0 1 0		0 1 0
0 0 0 0 0 0 1		0 0 0

(2)

for r = 1101110

$$Hr = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = [0\ 1\ 0]$$

⇒ error in bit # 6

⇒ d' = 1101100 ⇒ 1101 (2)

~~(c) $d_{min} = 3$ ⇒ $t = int\left(\frac{d_{min} - 1}{2}\right) = int\left(\frac{3-1}{2}\right) = 1$~~

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(Q.2)

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$ (1)

(a) $$\Rightarrow G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$ (2)

(b) $c = dG$

d	c
0 0 0	0 0 0 0 0 0
0 0 1	0 0 1 1 0 1
0 1 0	0 1 0 1 1 1
0 1 1	0 1 1 0 1 0
1 0 0	1 0 0 1 1 0
1 0 1	1 0 1 0 1 1
1 1 0	1 1 0 0 0 1
1 1 1	1 1 1 1 0 0

(2)

(c) $d_{min} = 3 \Rightarrow t = \text{int}\left(\frac{3-1}{2}\right) = 1$ error to be correct (2)

(d)

0 0 0 0 0 0	0 0 0
1 0 0 0 0 0	1 1 0
0 1 0 0 0 0	1 1 1
0 0 1 0 0 0	1 0 1
0 0 0 1 0 0	1 0 0
0 0 0 0 1 0	0 1 0
0 0 0 0 0 1	0 0 1

(2)

(e) $$Hr = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = [1\ 1\ 1]$$ (1)

$\Rightarrow$ error on the 2^{nd} bit $\Rightarrow$ 1 1 1 1 0 0

3

Q.3 $g(x) = x^3 + x^2 + 1 \Rightarrow 1101$

(a)

```
1101 ) 1000000
       1101
       0101000
        1101
        011100
         1101
         00110  => 1000110
```

(2)

(b)

```
1101 ) 1000110
       1101
       0101110
        1101
        011010
         1101
         000000  => ok! no errors.
```

(1)

(c)

```
1101 ) 1001110
       1101
       0100110
        1101
        010010
         1101
```

(1)

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